REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
12/02/94

3. REPORT TYPE AND DATES COVERED
Final 1 May 91 - 31 Oct 94

4. TITLE AND SUBTITLE
Systems of Hyperbolic Partial Differential Equations

5. FUNDING NUMBERS
DAAL03-91-G-0122

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8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U.S. Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
ARO 28702.5-MA

11. SUPPLEMENTARY NOTES
The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

This description of results from the project covers research in two areas:

1. Wave propagation for equations describing elastoplastic deformations of granular materials.
2. Hyperbolic conservation laws.

14. SUBJECT TERMS
Elastoplasticity, hyperbolic equations

15. NUMBER OF PAGES
4

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102

SYSTEMS OF HYPERBOLIC PARTIAL DIFFERENTIAL EQUATIONS

FINAL REPORT

MICHAEL SHEARER

DECEMBER, 1994

U.S. ARMY RESEARCH OFFICE

GRANT NUMBER DAAL03-91-G-0122

NORTH CAROLINA STATE UNIVERSITY

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Summary of Research Results

This description of results from the project covers research in two areas:

1. Wave propagation for equations describing elastoplastic deformations of granular materials.

2. Hyperbolic conservation laws.

1. PLASTIC FLOW

This part of the project was carried out in collaboration with David Schaeffer of Duke University. Our research focused on issues of wave propagation in the equations of motion of elastoplasticity. The partial differential equations are derived from conservation of mass and momentum, augmented by constitutive laws that relate the dependent variables algebraically. The constitutive laws considered describe properties of granular materials.

Scale-invariant problems.

We used scale-invariant initial-value problems (i) to clarify an example of nonuniqueness in multidimensional nonassociative plasticity [1] and (ii) to construct solutions for certain model problems in plasticity [3].

Shear bands

References [5] and [4] solve the Riemann problem for a system based on antiplane shearing, for cases in which strong loading forces a shear band to form. The shear band is treated as a discontinuity in the material subject to an ad hoc jump condition. Mathematically the solution with a shear band is interesting because (i) the Riemann problem does not admit a scale-invariant solution and (ii) the generalized solution leads to a natural free boundary problem for the wave equation. Reference [5] analyzed the small-time asymptotics of the problem; [4] proves a rigorous existence theorem. The latter was surprisingly tricky, apparently requiring the Nash-Moser theorem. These analytic results have played a major role in guiding computations with shear bands undertaken by my colleague Xavier Garaizar.

Incrementally nonlinear constitutive laws

References [8] and [9] begin the theoretical analysis of wave propagation with rate-independent, incrementally nonlinear constitutive laws, a class which contains both yield-vertex models and hypoplastic models. Physically these are important in improving the accuracy of modeling, especially in the presence of large stress rotations, and mathematically they are interesting because the resulting partial differential equations are fully nonlinear. These papers demonstrate that, for fully nonlinear equations, there is a subtle relationship between linear well-posedness (i.e., hyperbolicity) and nonlinear well-posedness (existence of a unique solution of initial value problems and continuous dependence on the data). In reference [8], we analyze the structure of discontinuous solutions and identify a class of 2x2 model systems, for which well-posedness of the Riemann problem can be identified with the

property of linear hyperbolicity. In reference [9] we present examples to show that such an identification is not possible for all models. In this paper we also study solutions of the regularized equations, deriving the structure anticipated in [8]. Reference [12] studies numerically the blow-up of the solution in an ill-posed case. In recent work [11], we are extending the analysis to the full 5x5 system for hypoplastic plane waves in two dimensions. A preliminary account of this work appears in [10].

2. HYPERBOLIC CONSERVATION LAWS

There are classes of nonstrictly hyperbolic systems for which the characteristic speeds are real, but coincide along a curve. In [2], we give a local analysis and classification of such equations, and show how new types of shocks, known as *singular shocks*, are a crucial part of solving Riemann problems. We give a detailed asymptotic analysis of corresponding solutions of the regularized, parabolic, equations. These solutions blow up at a single point as the dissipation approaches zero.

In [7], based on the masters project of my student, I show how undercompressive shocks are the limit of travelling wave solutions of a scalar hyperbolic conservation law with added dissipative and dispersive terms. In this paper, the Riemann problem is solved using the new shock waves, and numerical results are given that show clearly the stability of the travelling waves. The analysis depends on new results concerning saddle–to–saddle trajectories for cubic vector fields in the plane.

A much more complicated analysis was performed earlier [6] for a p-system of mixed type, forming the thesis of Yadong Yang. The paper demonstrates precisely the regime of initial data for which admissible solutions are nonunique.

PUBLICATIONS

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4. (with D.G. Schaeffer) Unloading near a shear band: a free boundary problem for the wave equation. Comm. P.D.E., **18** (1993), 1271-1298.

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9. (with D.G. Schaeffer) Fully nonlinear hyperbolic systems of partial differential equations related to plasticity. Comm. P.D.E., to appear.

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11. (with D.G. Schaeffer) Riemann problems for 5×5 systems of fully nonlinear equations related to hypoplasticity. Preprint.

12. S. Myagchilov, *A class of model equations related to hypoplasticity.* M.Sc. project report, N.C. State University, 1994.

PERSONNEL SUPPORTED

Yadong Yang. Ph.D. September, 1991.